Mathematical Economics

# Linear Demand & Supply Functions

Typical Demand Function

- Demand for a good can be expressed using mathematical functions

- A typical demand function looks like $Q_D = a - bP$ where,

  - Q_D represents the quantity demanded

  - **a** represents the autonomous level of demand, or the quantity demanded if the price were zero (**Q-intercept**)

  - **b** represents the change in quantity demanded resulting from a change in price (the slope calculated as $\Delta Q_d / \Delta P$)

  - **P** represents the price of a single item

Example; Demand for Cappuccinos

- Suppose the demand for cappuccinos in Richmond Hill can be expressed as $Q_D = 600 - 50P$

  - It is possible to construct both a demand schedule and demand curve from this demand function

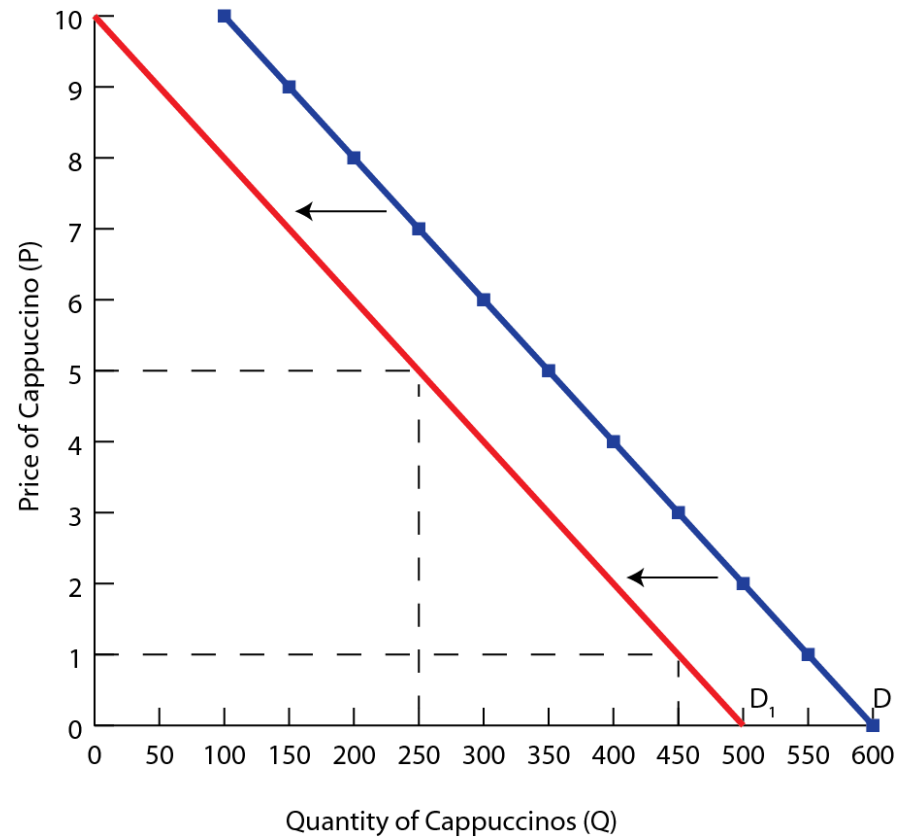| Linear demand schedule: Cappuccinos	
Price of Cappuccinos (P)	**Quantity demanded per day (Q_D)**
10	100
8	200
6	300
4	400
2	500
0	600

Quantity of Cappuccinos (Q)
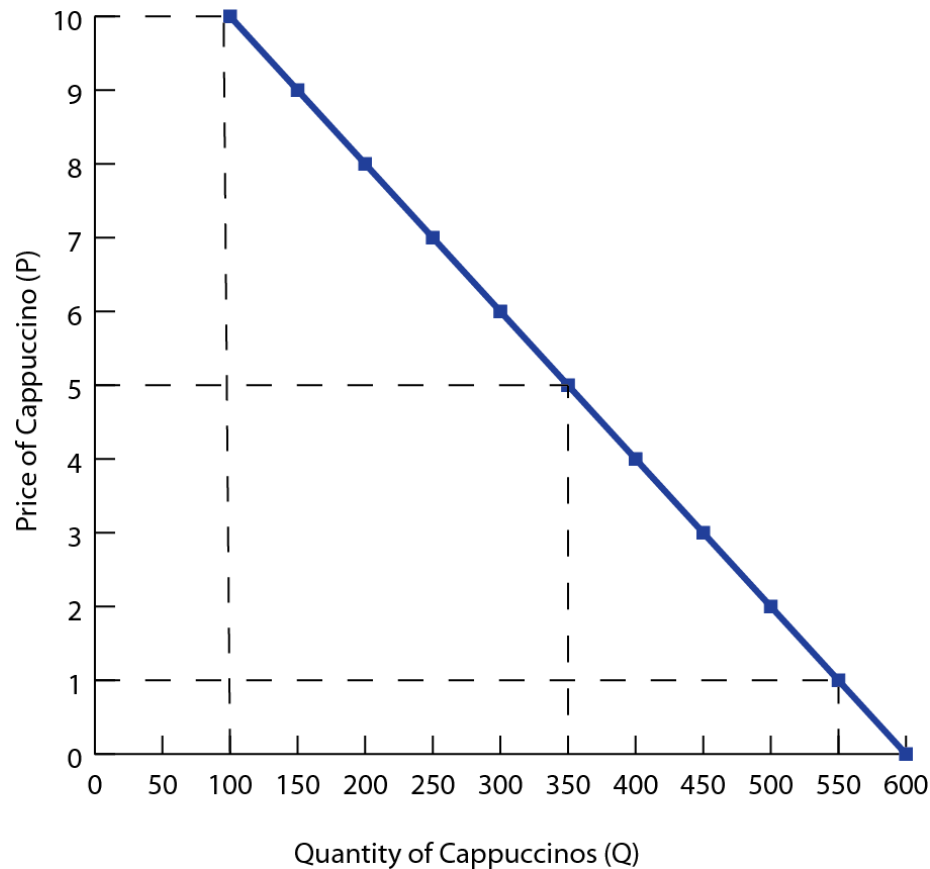
o A movement along the demand curve will occur any time the price of cappuccinos increases or decreases

o At lower prices, more are demanded; at higher prices fewer cappuccinos are demanded by consumers

Changes in 'a'

- If any of the determinants of demand change, then the '**a**' value in the demand function will change

  o The demand curve will shift either left or right

- **Example;** Suppose the demand function changes to $Q_D = 500 - 50P$

Linear demand schedule: Cappuccinos	
Price of Cappuccinos (P)	**Quantity demanded per day (Q_D)**
10	0
8	100
6	200
4	300
2	400
0	500

The graph shows Price of Cappuccino (P) on the vertical axis (0 to 10) and Quantity of Cappuccinos (Q) on the horizontal axis (0 to 600). Two demand curves are shown: the original curve D (blue) with P-intercept at 10 and Q-intercept at 600, and the new curve D_1 (red) with P-intercept at 10 and Q-intercept at 500. Arrows indicate a leftward shift.

- o   The slope of the new demand curve will be the same as the original curve

- o   The demand curve shifts left, or down by 100 units at every price

- o   The Q-intercept is now at 500 rather than 600

Changes in 'b'

- Changes to the price coefficient **b** will change the steepness of the demand curve

  o This changes the price elasticity of the demand curve

- **Example;** Suppose the demand function changes to $Q_D = 600 - 30P$

Linear demand schedule: Cappuccinos	
Price of Cappuccinos (P)	**Quantity demanded per day (Q_D)**
10	300
8	360
6	420
4	480
2	570
0	600
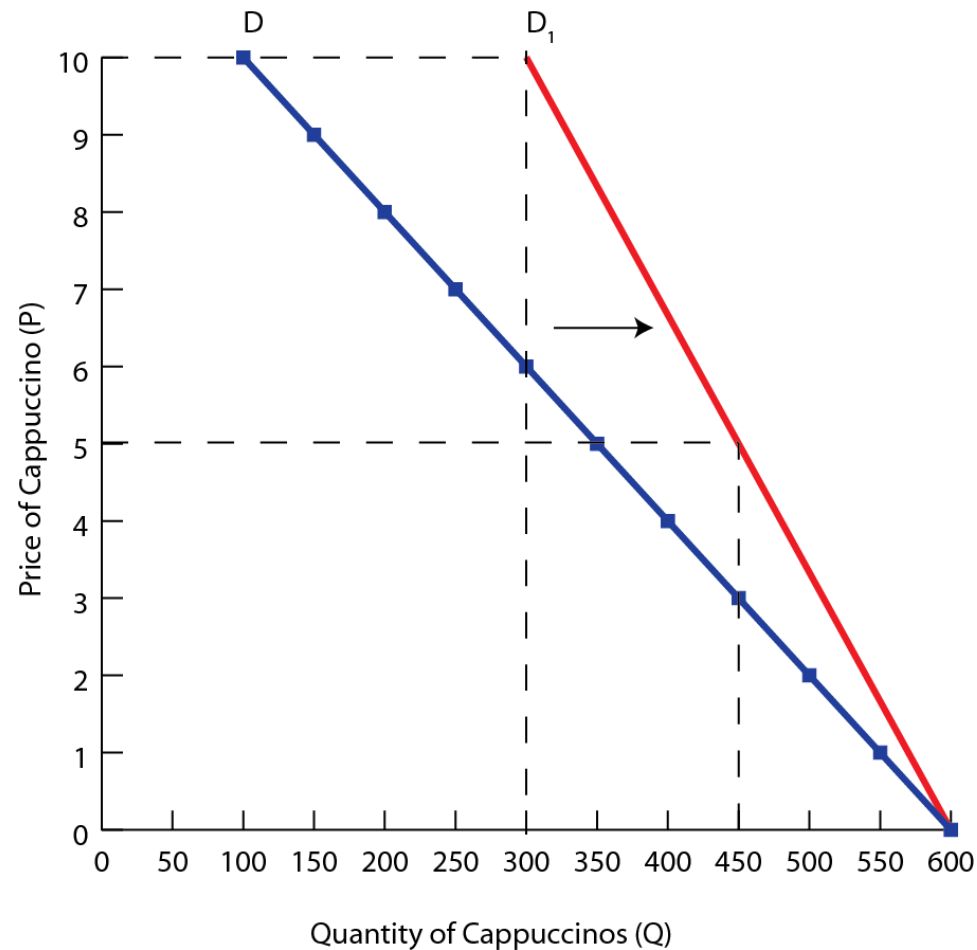
o The demand curve has become steeper, indicating consumers are less sensitive to price changes than previously

o The overall demand for cappuccinos has become more inelastic

Typical Supply Function

- Supply for a good can also be expressed using mathematical functions

- A typical supply function looks like $\mathbf{Q_S = c + dP}$ where,

    - $\mathbf{Q_S}$ represents the quantity supplied

    - $\mathbf{c}$ represents the autonomous level of supply, or the quantity produced if the price were zero (**Q-intercept**)

    - $\mathbf{d}$ represents the rate at which a change in price will cause the quantity supplied to increase (the slope calculated as $\Delta\mathbf{Q_s}/\Delta P$)

    - $\mathbf{P}$ represents the price of a single item

Example; Supply Cappuccinos

- Suppose the supply of cappuccinos in Richmond Hill can be expressed as
  $Q_s = -200 + 150P$

  o It is possible to construct both a supply schedule and supply curve from this supply function

| Linear supply schedule: Cappuccinos	
Price of Cappuccinos (P)	Quantity supplied per day (Q_D)
10	1300
8	1000
6	700
4	400
2	100
0	-200
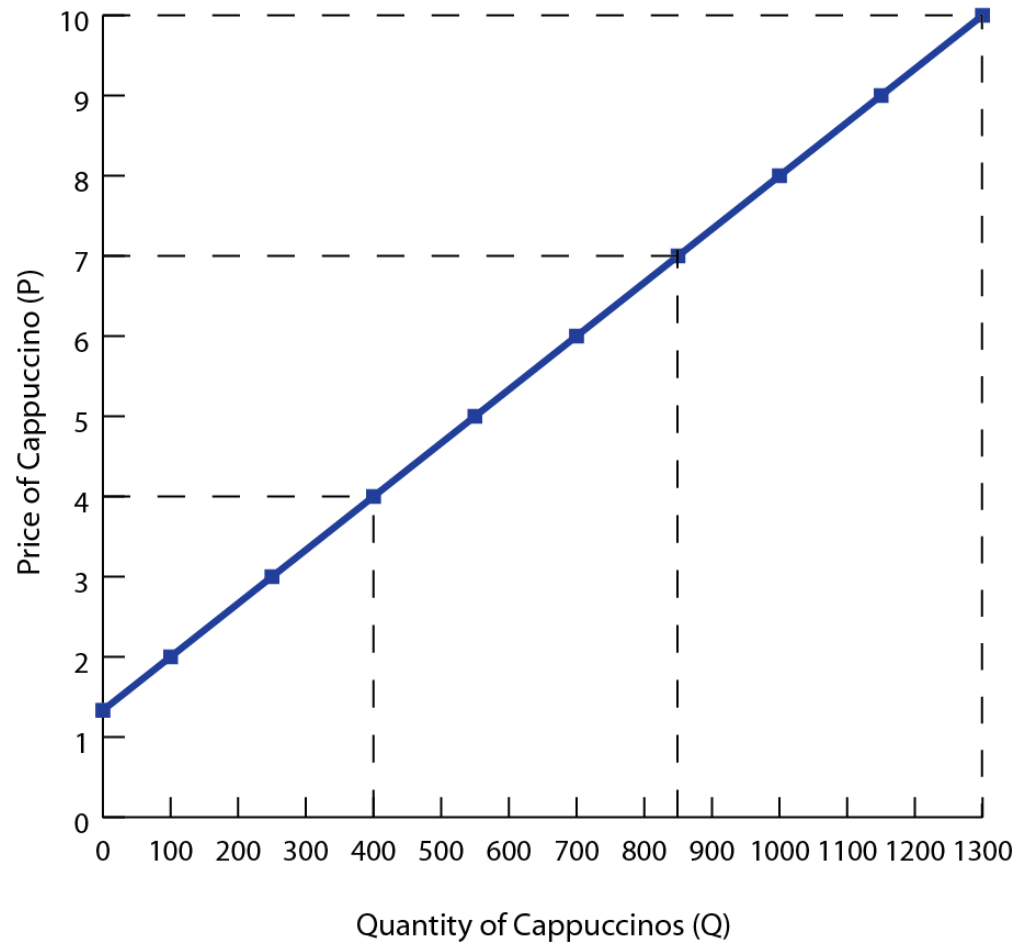
o There is a direct relationship between the price and quantity supplied. As price falls, producers are willing to provide fewer drinks to the market

Changes in 'c'

- If any of the non-price determinants of supply change, then the '**c**' value in the supply function will change

  o The supply curve will shift either left or right

- **Example;** Suppose the supply function changes to $Q_S = -100 + 150P$

| Linear supply schedule: Cappuccinos	
Price of Cappuccinos (P)	**Quantity supplied per day (Q_D)**
10	1400
8	1100
6	800
4	500
2	200
0	-100
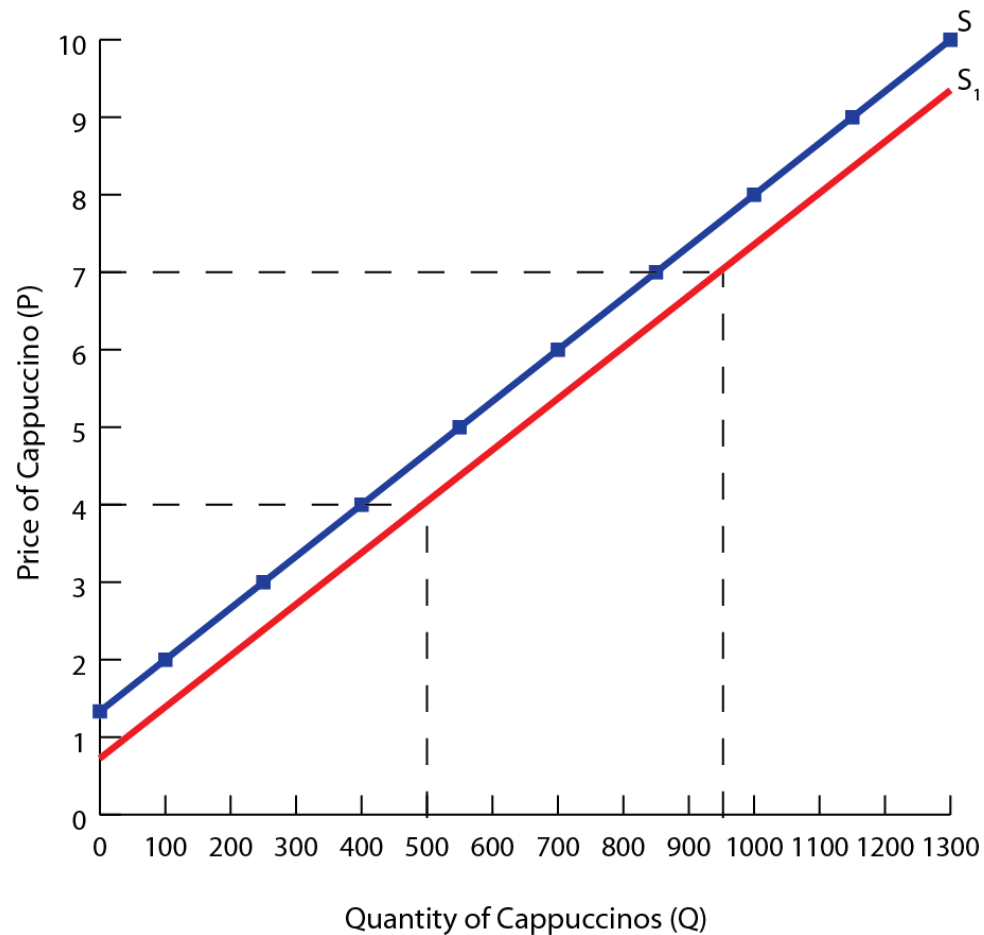
o The quantity supplied at each price level is reduced, but the slope remains the same

o The price intercept is now $0.75 rather than $1.33

Changes in 'd'

- Changes to the price coefficient **d** will change the steepness of the supply curve

  - This changes the price elasticity of the supply curve

- **Example;** Suppose the supply function changes to **Q = -200 + 200P**

Linear supply schedule: Cappuccinos	
Price of Cappuccinos (P)	**Quantity supplied per day (Q_D)**
10	1800
8	1400
6	1000
4	600
2	200
0	-200
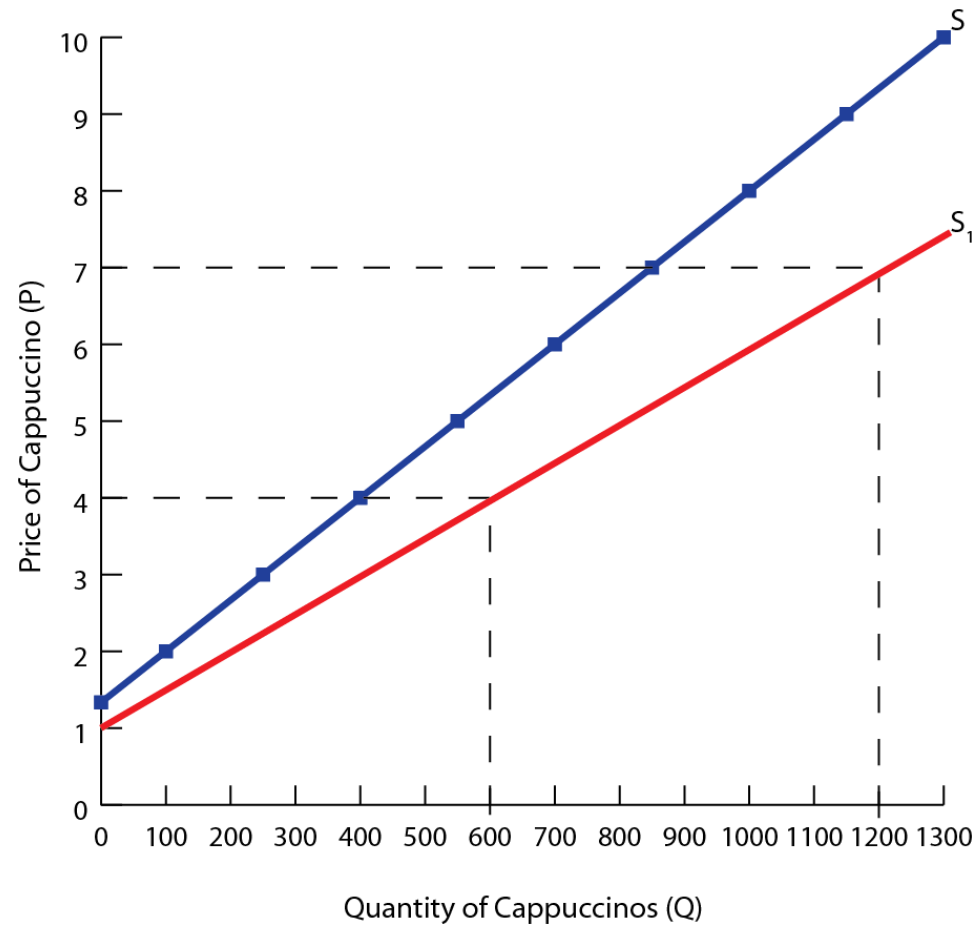
- The supply curve is less steep, indicating consumers are more sensitive to price changes than previously

- The overall supply for cappuccinos has become more elastic

Summary

	Component	Change	Impact on Demand/Supply
Linear Demand $(Q_d = a - bP)$	**a** or **c**	Increase	Rightward shift (Increase)
	a or **c**	Decrease	Leftward shift (Decrease)
Linear Supply $(Q_s = c + dP)$	**b** or **d**	Increase	Less steep (Elastic)
	b or **d**	Decrease	More steep (Inelastic)

Study Questions

o **1.** Use the linear demand function, $Q_D = 300 - 30P$

o **A.** Create a demand schedule with prices of $0, $3, $5, $7 and $9

o **B.** Create a demand curve, plotting point from your demand schedule

o **C.** Decrease the value of **a**, the autonomous element of demand, by 30 units. Create a new demand schedule, with the adjusted values for Q_D.

o **D.** On your previous diagram, show the new demand curve

o **E.** Now change the value of the price coefficient, in the original function to -10. Calculate the prices and quantities demanded, and list them of a demand schedule

o **F.** Create a new demand curve

Study Questions

o **1.** Use the linear demand function, $Q_S = -100 + 10P$

o **A.** Create a supply schedule with prices of $10, $20, $30, $40 and $50

o **B.** Create a supply curve, plotting point from your demand schedule

o **C.** Decrease the value of **c**, the autonomous element of supply, by 50 units. Create a new supply schedule, with the adjusted values for Q_S.

o **D.** On your previous diagram, show the new supply curve